

# LONGITUDINAL STABILITY OF SHORT BUNCHES IN STORAGE RINGS WITH STRONG LONGITUDINAL FOCUSING\*

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## Abstract

In the BESSY VSR project, the variable bunch length storage ring, two high gradient accelerating structures at 1.5 and 1.75 GHz will be phased such that long and short bunches can be stored simultaneously. The longitudinal stability of the short bunches is investigated taking into account the shielded CSR- and a purely inductive impedance. Multi particle tracking studies and numerical solutions of the Vlasov-Fokker-Planck equation show that threshold currents for short bunches do not follow the simple scaling law which was found for long bunches. The inductive impedance can even lower the thresholds for the instability. With an 80 times increased accelerating gradient and reasonable assumptions on the inductive impedance for shorter bunches stable operation can be expected with bunches 1.8 ps long (RMS-value) and 0.8 mA current. According to the calculations and operating in a dedicated low- $\alpha$  mode will produce stable 40  $\mu$ A bunches with 400 fs length.

## INTRODUCTION

At the 3<sup>rd</sup> generation synchrotron radiation light source BESSY II shorter light pulses are requested by a growing number of users. Laser slicing is operated routinely and produces pulses as short as 100fs FWHM. Operating in the short bunch, low- $\alpha$  mode delivers bunches shorter than 6 ps FWHM. This is offered for up to 3 weeks per year. Slicing and low- $\alpha$  mode suffer both from a too low intensity. One way to increase the intensity of light pulses, or overcome the quite low stable bunch currents, is the increase of the longitudinal RF-focusing. This produces shorter bunches and shifts threshold currents to higher values. A neat way to produce short bunches without sacrificing flux or brilliance for other users was proposed by G. Wüstefeld, et al. [1]: long and short bunches can be stored simultaneously with proper phasing of two superconducting high gradient accelerating structures operating at 1.5 and 1.75 GHz, 3 and 3.5 times the fundamental 500 MHz-RF-system. Most of the charge will be stored in long bunches producing most of the photon flux and, at the same time, a few short bunches carrying charge just below their instability threshold will emit the desired short light pulses. The purpose of this paper is to analyse the longitudinal stability of these short bunches.

## MODEL OF SHIELDED CSR

For very short bunches the coherent synchrotron radiation (CSR) is dominating the longitudinal beam dynamics. In the calculations presented here the model of

the steady-state CSR theory for bunches moving on a circle with radius  $\rho$  is used. This is justified by the fact that the BESSY dipoles are long enough so that the light emitted from a trailing electron at the entrance of the dipole can overtake the bunch and interact with the electrons at the head of the bunch [2] already inside the dipole magnet. The slippage length is a factor of 3 to 14 longer than the length of the investigated bunches. Bunches are not yet short enough to assume that the interaction takes place in free-space. Shielding effects of the nearby dipole vacuum chamber have to be taken into account. This is done with the wake function derived by J. Murphy, et al. [3] for the model of two perfectly conducting infinite parallel plates separated by  $2h$ .

Some of the results of the simulations and also the observations are easier to understand within the impedance picture of the shielded CSR-interaction first derived by R. Warnock [4] and later expressed in scaled parameters by Y. Cay [5]. The resulting CSR-impedance is presented in Figure 1. The plates suppress the impedance at low frequencies and at higher frequencies the impedance approaches the free-space values. In between a resonance-like enhancement appears. The frequency of the maximum as deduced from the Olver expansion of the impedance [6] is located around  $n_{\max} = \omega/\omega_0 = \omega\rho/c = 2/3^{1/2}(\pi\rho/2h)^{3/2}$ . This broad resonance has features of a broad-band resonator and can create similar instabilities.

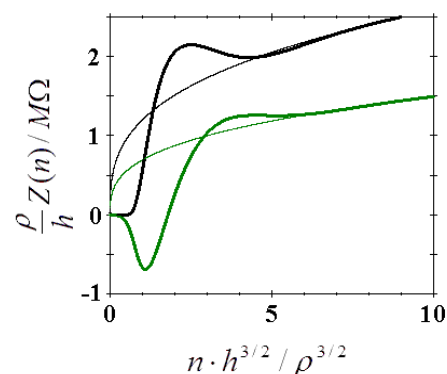


Figure 1: Scaled real part (black) and imaginary part (green) of the CSR impedance in free space (thin lines) and shielded by two infinite perfectly conducting parallel plates (thick lines).

Based on the shielded CSR-model, K. Bane, et al. [7] found a very simple scaling law valid for a large range of parameters, except for bunch lengths with  $1/\sigma$  in the neighbourhood of this resonance. In this region the strong instability turns into a weak instability and threshold currents are dramatically reduced. BESSY VSR will produce bunches with a length in the critical region.

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Therefore, detailed numerical studies were performed in order to investigate the longitudinal stability of these bunches in more detail. Stability was analysed numerically with the help of a Vlasov-Fokker-Planck solver [8] or by tracking many particles. Results of these calculations together with experimental results [9] are presented for BESSY II in Figure 2. In the simulations the bunch length is varied either by changing the momentum compaction factor  $\alpha$  and keeping the RF-voltage fixed (the green curve) or, like in the BESSY VSR case, by increasing the RF-Voltage at fixed  $\alpha$  value (the black curve). Bane’s simple scaling law and thus the underlying scaling properties for BESSY VSR, that for a given bunch length the ratio of threshold current divided by the applied RF-voltage is constant, appears to be valid only for bunches longer than 3 ps. For shorter bunches the weak instability leads to thresholds being dependent on the product of the synchrotron frequency and the longitudinal damping time. The measured thresholds shown as red squares are in fair agreement with the green curve and are clearly systematically shifted to higher currents for the longer bunches.

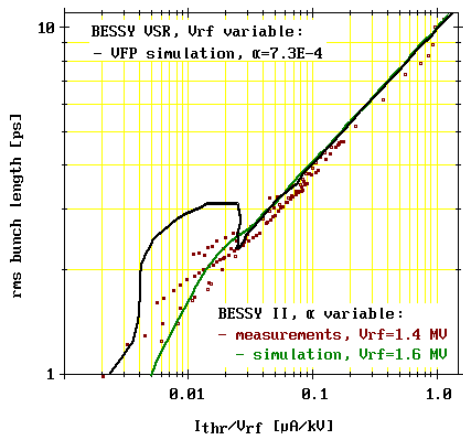


Figure 2: Comparison of predicted and observed instability thresholds.

Good agreement is also obtained for the frequency of the lowest unstable mode at BESSY II and for the instability threshold currents observed at the MLS where the region of the weak instability was covered and the thresholds behaved as predicted [10]. However, the bunch lengthening observed with a streak camera before the instability sets in cannot be explained by the shielded CSR-model alone.

### ADDITION OF INDUCTIVE IMPEDANCE

This bunch lengthening is usually attributed to the primarily inductive impedance of the vacuum chamber of the storage ring. The simplest model for this impedance is  $Z = -iL \cdot \omega = -iL \cdot n \cdot \omega_0$  with a bunch length dependent inductive part  $L$ . Good agreement with the BESSY II bunch length measurements is obtained with  $|Z/n| = 0.2 \dots 0.35 \Omega$  for bunches with a length between 2 and 15 ps [11]. For the shorter bunches as produced by BESSY VSR

one expects  $|Z/n| < 0.2 \Omega$ . The corresponding delta-function wake for such an impedance was added to the codes and in Figure 3 the resulting predictions are shown for BESSY VSR operated with the standard value of the momentum compaction factor.  $|Z/n|$  is extrapolated and expected to be around  $0.1 \Omega$  – exact values are difficult to obtain because the spectrum of the shortest bunches extends up to the THz region and impedance calculations for the complete vacuum chamber cannot be performed with existing codes.

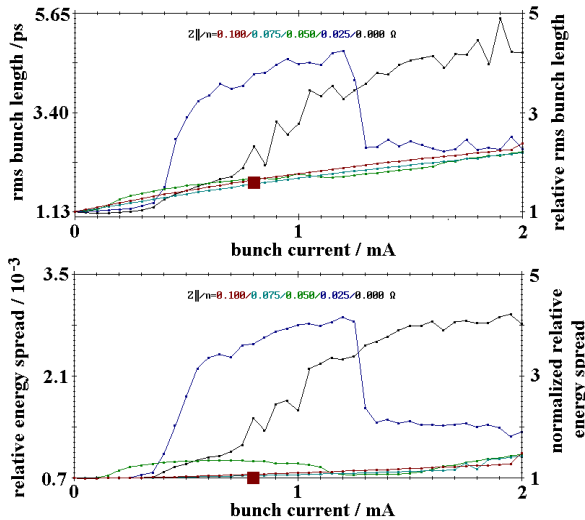


Figure 3: Results of multi particle tracking simulations with shielded CSR and inductive impedance. The red squares are the prediction for BESSY VSR. Bunches with an rms-length of 1.7 ps should be stable up to 0.8 mA. An inductive part larger than  $0.075 \Omega$  would shift the threshold current to even higher values.

The threshold current is defined as the current where energy spread suddenly increases. The slow increase found in the multi particle tracking studies is a numerical artefact introduced by the  $\delta$ -function wake. Usually values found from the numerical solution of the Vlasov-Fokker-Planck (VFP) equation agree with the results from multi particle tracking calculations. For small values  $|Z/n|$  the threshold is reduced – clearly seen in the increased energy spread at lower bunch current. With  $|Z/n|=0.025 \Omega$  bunches suffer from energy widening and bunch lengthening at quite low beam current. In this regime the bunches show the features of a kind of “binary star instability” as already found in the instability analysis of the broad-band resonator impedance by M. D’yachkov and R. Baartman [12]. Figure 4 presents the normalized longitudinal phase space for this situation as a function of beam current. In this regime the instability is saw tooth like but most of the time the bunch is split into two parts rotating around each other. Opposite to the standard instability of that type, beam parameters like bunch length and energy spread increase slowly by developing the binary structure until the structure smears out and making the bunch to shrink rather fast. At higher current the

instability calms down most likely due to the increased tune spread among the electrons.

The multi particle technique is superior for bunch currents far above the instability threshold. For BESSY VSR short and very intense bunches are requested for laser slicing. Particle tracking delivers insight into the details of the dynamics of these bunches much faster than the solution of the VFP equation.

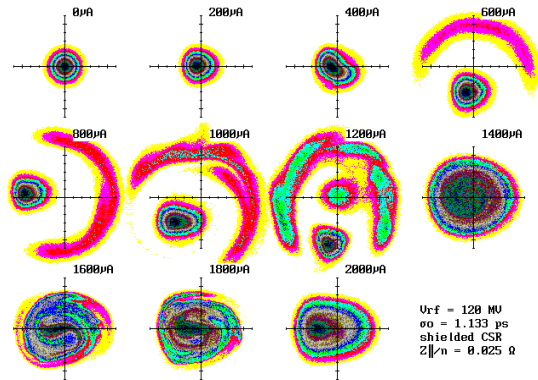


Figure 4: Normalized longitudinal phase space for  $|Z/n|=0.025\Omega$  from multi particle tracking simulations. The instability sets in with a quite regular motion of the bunch, followed by a splitting of the bunch into two parts and calming down at even higher current and turning into a more turbulent instability.

### Low Alpha Operation

This mode is also attractive for BESSY VSR since bunches with a FWHM of less than a ps can be expected. Therefore, this bunch length regime was investigated in more detail. The uncertainty of the inductive impedance for these short bunches is even larger than in the previous case and values for  $Z/n < 0.05\Omega$  were assumed. Results of tracking studies are presented in Figure 5 and they are similar to what was found for the longer bunches. Remember, in both cases bunches are below the resonance condition:  $2\pi \cdot F_{res} \cdot \sigma = n_{max} \cdot c\sigma/\rho < 1$  and, due to its sign, the inductive impedance at first counter acts the bunch shortening and compensates the tune spread among electrons. This reduces the threshold current for small values of  $|Z/n|$ . For larger values bunch lengthening and increased tune spread shift the threshold to higher current. With the assumed  $|Z/n|$  short bunches with an rms-length of 400fs should be stable up to 40μA. Ideally, if the inductive impedance would be tuneable one could find the right compromise between a still stable and not too much lengthened bunch.

The absolute value of the inductive impedance at high frequency, usually assumed to fall off at least as  $\omega^{-1/2}$ , introduces the largest uncertainty in the prediction of the BESSY VSR threshold currents for short bunches. With the simple model used for the present calculations the impedance would be proportional to  $\omega$ . A better impedance model, including the additional contributions from the super conducting cavity strings, would allow for more accurate prediction.

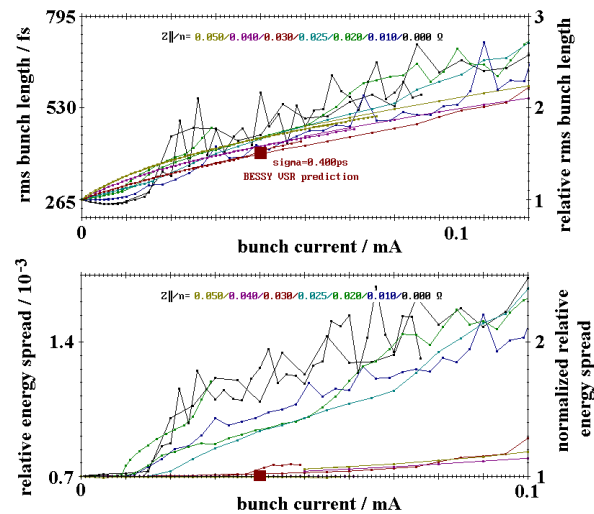


Figure 5: Current dependence of bunch length and energy spread of a short bunch in low- $\alpha$  mode.

## CONCLUSION

With reasonable assumptions on the imaginary or inductive part of the longitudinal impedance it can be predicted that BESSY VSR with an 80 times larger longitudinal accelerating gradient will deliver stable bunches with rms-length of 1.7ps at 0.8mA current under normal operating conditions and in low- $\alpha$  mode 400fs with 40μA current should be reached.

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